Semi-Inclusive $B \to K(K^*)X$ Decays with Initial Bound State Effects

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(November, 2000)

Abstract

The effects of initial b quark bound state for the semi-inclusive decays $B \to K(K^*)X$ are studied using light cone expansion and heavy quark effective theory methods. We find that the initial bound state effects on the branching ratios and CP asymmetries are small. In the light cone expansion approach, the CP-averaged branching ratios are increased by about 2% with respect to the free b-quark decay. For $\bar{B}^0 \to K^-(K^{*-})X$, the CP-averaged branching ratios are sensitive to the phase γ and the CP asymmetry can be as large as 7% (14%), whereas for $B^- \to \bar{K}^0(\bar{K}^{*0})X$ the CP-averaged branching ratios are not sensitive to γ and the CP asymmetries are small ($< 1\%$). The CP-averaged branching ratios are predicted to be in the ranges $(0.53 \sim 1.5)\times 10^{-4}$ $[(0.25 \sim 2.0) \times 10^{-4}]$ for $\bar{B}^0 \to K^-(K^{*-})X$ and $(0.77 \sim 0.84) \times 10^{-4}$ $[(0.67 \sim 0.74)\times 10^{-4}]$ for $B^- \to \bar{K}^0(\bar{K}^{*0})X$, depending on the value of the CP violating phase γ. In the heavy quark effective theory approach, we find that the branching ratios are decreased by about 10% and the CP asymmetries are

not affected. These predictions can be tested in the near future.

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I. INTRODUCTION

There have been considerable experimental and theoretical efforts to understand the properties of B decays. These studies have provided important information about the mechanism for B decays and the origin of CP violation. In the next few years large quantities of experimental data on B decays will become available. It is hoped that one will obtain even more important information in understanding the mechanism for B decays and the mechanism for CP violation. In particular, charmless hadronic B decays have played an important role in the determination of the CP violating parameter γ in the Standard Model (SM) [1–5]. While most of the studies have concentrated on the exclusive B decay modes for CP violation, there are also some studies for semi-inclusive decays [2,3]. At the quark level the relevant Hamiltonian for B decays in the SM is well understood. The major uncertainties for these decays come from our insufficient understanding of the long distance strong interaction dynamics involved in these decays. There are several methods which have been used to estimate the decay amplitudes, including naive factorization, QCD improved factorization and methods based on symmetry considerations.

Recently it has been argued that in the heavy quark limit, factorization is a good approximation [4] and several processes have been calculated [5]. Leading QCD corrections to the naive factorization can be studied for exclusive decays in a systematic way. In the calculation of exclusive decays, the hadronic matrix elements can be factorized and strong interaction dynamics can be parameterized into the relevant decay constants, light cone distribution amplitudes and transition form factors. At the present time, the light cone distribution amplitudes and transition form factors are not well known which introduce uncertainties in the calculations. Of course one should keep in mind that there may be large corrections of order Λ_{QCD}/m_b which needs further study. From quark hadron duality consideration, inclusive decays can be represented by quark level calculations and the uncertainties may be small. It is believed that theoretical calculations for exclusive decays contain more uncertainties than inclusive decays. Of course when going completely inclusive, there are less information

that can be extracted about strong and weak interaction dynamics and CP violation, and it is experimentally hard to identify final states inclusively. In this paper we will take the way in between by studying semi-inclusive decays following Ref. [2] in the hope that one may be able to reduce some of the hadronic uncertainties in exclusive decays on one hand, and still be able to obtain important information about B decays and CP violation with clear experimental signal on the other. We will study the charmless semi-inclusive decays $B \to KX$ and $B \to K^*X$. Here the X indicates states containing no charmed particles.

The decay modes $B \to K(K^*)X$ have been studied before [2,3]. In previous studies, several effects were treated phenomenologically, such as the number of colors was taken as an effective number and treated as a free parameter, the gluon virtuality q^2 in the penguin diagrams was assumed to be around $m_b^2/2$, and the bound state effects of b-quark inside the B meson was modeled by assuming its momentum to obey a Gaussian distribution. To have a better understanding of these decays, it is necessary to carry out calculations in such a way that the phenomenological treatments can be improved with better theoretical understanding. It has recently been shown that it is indeed possible in the heavy quark limit to handle most of the problems in exclusive B to two light meson decays from QCD calculations [4]. We will use the same formalism in our study of semi-inclusive decays in the factorization approximation, paying particular attention to the initial bound state effects.

The problems treated in the case of exclusive decays are different in some ways from the semi-inclusive decays studied here. The problems associated with the number of colors and the gluon virtuality can be treated the same way, but the initial b quark bound effects in semi-inclusive decays arise in different form from those in exclusive decays. In the exclusive decay case, the b quark bound state effects are taken care by decay constants and transition form factors. In the semi-inclusive case, there are contributions which, in the free quark decay approximation, can be viewed as a b quark decay into a meson and another quark. One needs to treat initial b quark bound state effects on more theoretical ground. This will be the main focus of this paper. We will study this problem using two different methods with one based on light cone expansion and another based on heavy quark effective theory.

To further reduce possible uncertainties associated with form factors, we will choose processes which have the least numbers of hadronic parameters beside the ones related to the initial bound state effects. We find that the following processes are particularly good for this purpose,

$$\begin{aligned} &\bar{B}^0 \to K^- X, \quad B^- \to \bar{K}^0 X, \\ &\bar{B}^0 \to K^{*-} X, \quad B^- \to \bar{K}^{*0} X. \end{aligned} \tag{1}$$

For these processes, the transition form factors for $B \to K$ and $B \to K^*$ do not show up in the factorization approximation because the bi-quark operator $\bar{s}\Gamma b$ (here Γ is some appropriate Dirac matrices) does not change the electric charge of the initial particle B and the final particle $K(K^*)$. Therefore for these processes there are only the $K(K^*)$ decay constants and parameters related to the initial bound state effects if small annihilation contributions are neglected.

The paper is arranged as follows. In Section II, we will study the decay amplitudes in the SM for the semi-inclusive $B \to K(K^*)X$ decays. In Section III, we will study the light cone and heavy quark effective theory formulation of the initial bound state effects on these semi-inclusive decays. And in Section IV, we will carry out numerical analyses of the energy spectra of the $K(K^*)$, branching ratios and CP asymmetries in $B \to K(K^*)X$, and draw our conclusions.

II. DECAY AMPLITUDES IN THE HEAVY QUARK LIMIT

In this section we study the short distance decay amplitudes for semi-inclusive $B \to K(K^*)X$ decays. The effective Hamiltonian for charmless B decays with $\Delta S = 1$ at the quark level is given by

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* (c_1 O_1 + c_2 O_2 + \sum_{n=3}^{11} c_n O_n) + V_{cb} V_{cs}^* \sum_{n=3}^{11} c_n O_n \right\}. \tag{2}$$

Here O_n are quark and gluon operators and are given by

$$O_1 = (\bar{s}_i u_j)_{V-A}(\bar{u}_j b_i)_{V-A}, \quad O_2 = (\bar{s}_i u_i)_{V-A}(\bar{u}_j b_j)_{V-A},$$
$$O_{3(5)} = (\bar{s}_i b_i)_{V-A}\sum_{q'}(\bar{q}'_j q'_j)_{V-(+)A}, \quad O_{4(6)} = (\bar{s}_i b_j)_{V-A}\sum_{q'}(\bar{q}'_j q'_i)_{V-(+)A},$$
$$O_{7(9)} = \frac{3}{2}(\bar{s}_i b_i)_{V-A}\sum_{q'} e_{q'}(\bar{q}'_j q'_j)_{V+(-)A}, \quad O_{8(10)} = \frac{3}{2}(\bar{s}_i b_j)_{V-A}\sum_{q'} e_{q'}(\bar{q}'_j q'_i)_{V+(-)A},$$
$$O_{11} = \frac{g_s}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} G^a_{\mu\nu}\frac{\lambda_a^{ij}}{2}(1+\gamma_5)b_j, \tag{3}$$

where $(V \pm A)(V \pm A) = \gamma^\mu(1 \pm \gamma_5)\gamma_\mu(1 \pm \gamma_5)$, $q' = u, d, s, c, b$, $e_{q'}$ is the electric charge number of the q' quark, λ_a is the color SU(3) Gell-Mann matrix, i and j are color indices, and $G_{\mu\nu}$ is the gluon field strength.

The Wilson coefficients c_n have been calculated in different schemes [6]. In this paper we will use consistently the NDR scheme. The values of c_n at $\mu \approx m_b$ with the next-to-leading order (NLO) QCD corrections are given by [6]

$$c_1 = -0.185, \quad c_2 = 1.082, \quad c_3 = 0.014, \quad c_4 = -0.035, \quad c_5 = 0.009, \quad c_6 = -0.041,$$
$$c_7 = -0.002\alpha_{em}, \quad c_8 = 0.054\alpha_{em}, \quad c_9 = -1.292\alpha_{em}, \quad c_{10} = -0.263\alpha_{em}, \quad c_{11} = -0.143.$$

Here $\alpha_{em} = 1/137$ is the electromagnetic fine structure constant.

In order to make sure that the observed events are from rare charmless B decays, and other processes, such as $B \to D(D^*)X' \to K(K^*)X''$, do not contaminate the direct rare decay of $B \to K(K^*)X$ due to short distance interaction, we will make a cut on the $K(K^*)$ energy which will be set at $E_{K,K^*} > 2.1$ GeV. It has been shown that this cut can eliminate most of the unwanted events while leave most of the events induced by short distance contributions [2] because the matrix elements of the type $< K(K^*)|j_1|0 >< X|j_2|B >$ would results in a fast $K(K^*)$ in the final state. The resulting events will resemble two body type of decays with one of them be the $K(K^*)$ and another, back-to-back against the $K(K^*)$, will be X with small invariant mass M_X^2. With the cut $E_{K,K^*} > 2.1$ GeV, $M_X^2 < 5.7$ GeV^2.

The hadronic matrix element for a specific operator $< XK|O|B >$ is difficult to calculate at present. We will use factorization approximation to estimate it. The factorization approximation has been shown to hold in the heavy quark limit for exclusive B decays into two

light hadrons. The leading contribution for an operator which can be written as a product of two currents $j_1 = \bar{s}\Gamma_1 q'$ and $j_2 = \bar{q}'\Gamma_2 b$ with Γ_i carrying appropriate Lorentz and Dirac indices, $O = j_1 \cdot j_2$, is given by

$$\begin{aligned} < XK|O|B >_{fact} &= < K|j_1|0 >< X|j_2|B > + < X|j'_1|0 >< K|j'_2|B > \\ &+ < XK|j_1|0 >< 0|j_2|B > . \end{aligned} \tag{4}$$

The second term on the right-hand-side in the above represents the Fierz transformed factorization terms with $j'_1 = \bar{q}'\Gamma'_1 q'$ and $j'_2 = \bar{s}\Gamma'_2 b$. The third term is usually referred to as the annihilation contribution.

$B \to KX$ is a many-body decay, which is different from two-body decays. There are more ways of factorization for a many-body decay, such as $< X_1 K|j_1|0 >< X'_1|j_2|B >$ and $< X_2|j'_1|0 >< X'_2 K|j'_2|B >$, with $X = X_1 + X'_1 = X_2 + X'_2$. The three terms in Eq. (4) corresponding to the cases: $< X_1| =< 0|$, $< X'_2| =< 0|$ and $< X'_1| =< 0|$, respectively. For $B \to KX$ with a cut $E_K > 2.1$ GeV, the final state X has a small invariant mass. This is a quasi-two-body decay, with K and X moving rapidly apart in opposite directions. The probability of forming the final state $< X_1 K|$ with $< X_1| \neq< 0|$ is less than the probability of forming the simple final state $< K|$. This suggests that the contribution of the configuration $< X_1 K|j_1|0 >< X'_1|j_2|B >$ is dominated by $< K|j_1|0 >< X|j_2|B >$. Likewise, the contribution of the configuration $< X_2|j'_1|0 >< X'_2 K|j'_2|B >$ is dominated by $< X|j'_1|0 >< K|j'_2|B >$. The cases with $|X_1 >$ and $|X'_2 >$ not equal to $|0 >$ are also higher order in α_s and therefore α_s power suppressed. We will neglect them in our later discussions which also eliminate the third term in Eq. (4).

The above approximation is also supported by explicit calculation of the bremsstrahlung process, $b \to Kq'g$, which represents some of the α_s order corrections. It has been shown, in a similar situation of $b \to \phi s$ and $b \to \phi s g$, that the bremsstrahlung contributes less than 3% of the total branching ratio [7]. One can easily obtain from Ref. [7] an estimate of the contribution for the processes considered here. The bremsstrahlung contribution is small. Eq. (4) will adequately approximate the leading contributions and we will work with this

approximation.

In the heavy quark limit, a class of radiative corrections in powers of α_s, which does not change the form of the operators, can be included for the matrix elements. For a local operator the correction can be parameterized as the following, similar to the exclusive decays discussed in Refs. [4,5],

$$< XK|O|B >=< XK|O|B >_{fact} [1 + \sum_{n=1}^{\infty} r_n \alpha_s^n + O(\Lambda_{QCD}/m_b)], \tag{5}$$

where $< XK|O|B >_{fact}$ denotes the naive factorization result. $\Lambda_{QCD} \approx 0.3$ GeV is the strong interaction scale. The second and third terms in the square bracket indicate, respectively, higher order α_s and Λ_{QCD}/m_b corrections to the factorized matrix element.

Similar arguments can be made for $B \to K^* X$ decays also. For the $< K(K^*)|j_1|0 >< X|j_2|B >$ type, the decay amplitudes involves the $K(K^*)$ decay constants, while for the $< X|j_1'|0 >< K(K^*)|j_2'|B >$ type, it involves the transition form factors from B to $K(K^*)$, and the $< XK(K^*)|j_1|0 >< 0|j_2|B >$ type involves the B decay constant.

If all three terms in Eq. (4) contribute with the same order of magnitude, the accumulated uncertainties will be substantial due to large uncertainties in the transition form factors and the B decay constant. Fortunately we find that for $\bar{B}^0 \to K^-(K^{*-})X$ and $B^- \to \bar{K}^0(\bar{K}^{*0})X$, only the first and the third types of terms in Eq. (4) contribute due to electric charge conservation. This eliminates possible uncertainties from the transition form factors. Also as argued before the third term can be neglected because it is subleading and α_s power suppressed. There is only one term present, which considerably simplifies the calculation.

Using the effective Hamiltonian in Eq. (2), we obtain

$$\begin{aligned}
A(B \to KX) &= i\frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb}V_{qs}^* f_K [A^q P_K^\mu < X|\bar{q}'\gamma_\mu(1-\gamma_5)b|B > \\
&\quad + B^q < X|\bar{q}'(1-\gamma_5)b|B >], \\
A(B \to K^*X) &= \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb}V_{qs}^* m_{K^*} f_{K^*} \tilde{A}^q \epsilon_\lambda^{\mu *} < X|\bar{q}'\gamma_\mu(1-\gamma_5)b|B >,
\end{aligned} \tag{6}$$

where $q' = u$ and d for $\bar{B}^0$ and B^-, respectively. The decay constants are defined as $< K|\bar{s}\gamma^\mu(1-\gamma_5)q'|0 >= if_K P_K^\mu$ and $< K^*(\lambda)|\bar{s}\gamma^\mu(1-\gamma_5)q'|0 >= m_{K^*} f_{K^*} \epsilon_\lambda^{\mu *}$. We adopt the

standard covariant normalization $< B|B >= 2E_B(2\pi)^3\delta^3(\mathbf{0})$. The coefficients $A^q(\tilde{A}^q)$ and B^q are given by, for $\bar{B}^0 \to K^-(K^{*-})X$

$$
\begin{aligned}
A^q(\tilde{A}^q) &= a_1^q + a_4^q + a_{10}^q + a_{10a}^q, \\
B^q &= (a_6^q + a_8^q + a_{8a}^q)\frac{2m_{K^-}^2}{m_u + m_s}.
\end{aligned} \tag{7}
$$

For $B^- \to \bar{K}^0(\bar{K}^{*0})X$,

$$
\begin{aligned}
A^q(\tilde{A}^q) &= a_4^q - \frac{1}{2}a_{10}^q + a_{10a}^q, \\
B^q &= (a_6^q - \frac{1}{2}a_8^q + a_{8a}^q)\frac{2m_{K^0}^2}{m_d + m_s}.
\end{aligned} \tag{8}
$$

Including the lowest α_s order corrections in Eq. (5), a_i^q are given by

$$
\begin{aligned}
a_1^u &= c_2 + \frac{c_1}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}c_1 F_P, \\
a_1^c &= 0, \\
a_4^q &= c_4 + \frac{c_3}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}\left[c_3(F_P + G_P(s_s) + G_P(s_b)) + c_1 G_P(s_q)\right. \\
&\quad \left. + (c_4 + c_6)\sum_{f=u}^{b} G_P(s_f) + c_{11}G_{P,11}\right], \\
a_6^q &= c_6 + \frac{c_5}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}\left[c_3(G'_P(s_s) + G'_P(s_b)) + c_1 G'_P(s_q)\right. \\
&\quad \left. + (c_4 + c_6)\sum_{f=u}^{b} G'_P(s_f) + c_{11}G'_{P,11}\right], \\
a_8^q &= c_8 + \frac{c_7}{N}, \\
a_{8a}^q &= \frac{\alpha_s}{4\pi}\frac{C_F}{N}\left[(c_8 + c_{10})\frac{3}{2}\sum_{f=u}^{b} e_f G'_P(s_f) + c_9\frac{3}{2}(e_s G'_P(s_s) + e_b G'_P(s_b))\right], \\
a_{10}^q &= c_{10} + \frac{c_9}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}c_9 F_P, \\
a_{10a}^q &= \frac{\alpha_s}{4\pi}\frac{C_F}{N}\left[(c_8 + c_{10})\frac{3}{2}\sum_{f=u}^{b} e_f G_P(s_f) + c_9\frac{3}{2}(e_s G_P(s_s) + e_b G_P(s_b))\right],
\end{aligned} \tag{9}
$$

where $N = 3$ is the number of colors, $C_F = (N^2 - 1)/(2N)$, and $s_f = m_f^2/m_b^2$. The other items are given by

$$
F_P = -12\ln\frac{\mu}{m_b} - 18 + f_P^I,
$$

$$
\begin{aligned}
f_P^I &= \int_0^1 dx g(x)\phi_P(x), \quad g(x) = 3\frac{1-2x}{1-x}\ln x - 3i\pi, \\
G_P(s) &= \frac{2}{3} - \frac{4}{3}\ln\frac{\mu}{m_b} + 4\int_0^1 dx\phi_P(x)\int_0^1 duu(1-u)\ln[s-u(1-u)(1-x)-i\epsilon], \\
G_{P,11} &= -\int_0^1 dx\frac{2}{1-x}\phi_P(x), \\
G'_K(s) &= \frac{1}{3} - \ln\frac{\mu}{m_b} + 3\int_0^1 dx\phi_K^0(x)\int_0^1 duu(1-u)\ln[s-u(1-u)(1-x)-i\epsilon], \\
G'_{K,11} &= -\int_0^1 dx\frac{3}{2}\phi_K^0(x), \\
G'_{K^*}(s) &= 0, \\
G'_{K^*,11} &= 0,
\end{aligned}
\tag{10}
$$

where the subscript P can be K or K^*, indicating that the coefficients a_i^q are process dependent. $\phi_K(x)$ and $\phi_K^0(x)$ are the twist-2 and twist-3 kaon meson distribution amplitudes, respectively. $\phi_{K^*}(x)$ is the leading twist distribution amplitude for the longitudinally polarized K^*. In this paper we will take the following forms for them [5],

$$
\phi_{K,K^*}(x) = 6x(1-x), \quad \phi_K^0(x) = 1. \tag{11}
$$

The amplitudes in Eq. (6) are from perturbative QCD calculation in the heavy quark limit. The number of colors should not be treated as an effective number, but has to be 3 from QCD. The results are, in principle, renormalization scale and scheme independent. The problem associated with the gluon virtuality $k^2 = (1-x)m_B^2$ in the naive factorization calculation is also meaningfully treated by convoluting the x-dependence with the meson distribution amplitudes in the functions $G_P(s)$ and $G'_P(s)$.

III. INITIAL BOUND STATE EFFECTS

In this section we study the decay rates for $B \to K(K^*)X$, taking into account b quark bound state effects, using two different methods, the light cone expansion method and the heavy quark effective theory method.

We will work out, in detail, the formulation for $B \to KX$ in the following. The results for $B \to K^*X$ can be easily obtained in a similar way. Without taking into account the

initial bound state effects, that is in the free b quark decay approximation, the decay can be viewed as the two body process $b \to Kq'$ and one obtains [2]

$$\Gamma(B \to KX) \approx \Gamma(b \to Kq') = \frac{f_K^2}{8\pi}(m_b^2|\alpha|^2 + |\beta|^2)m_b,$$
$$\alpha = \frac{G_F}{\sqrt{2}}\sum_{q=u,c} V_{qb}V_{qs}^* A^q, \quad \beta = \frac{G_F}{\sqrt{2}}\sum_{q=u,c} V_{qb}V_{qs}^* B^q. \tag{12}$$

If the b quark mass is infinitively large, $Br(B \to K(K^*)X)$ is equal to $Br(b \to K(K^*)q')$. However due to initial b quark bound state effects there are corrections [8,9]. We now proceed to study the initial bound state effects on the decay rates.

The differential decay rate for $B \to KX$ in the B rest frame, following the procedure in Ref. [8], is given by

$$d\Gamma(B \to KX) = \frac{1}{2m_B}\frac{d^3\mathbf{P}_K}{(2\pi)^3 2E_K}\sum_X (2\pi)^4\delta^4(P_B - P_K - P_X)|A(B \to KX)|^2. \tag{13}$$

Using $\int d^4y \exp[-iy\cdot(P_B - P_K - P_X)] = (2\pi)^4\delta^4(P_B - P_K - P_X)$, we have

$$\begin{aligned}
&\sum_X (2\pi)^4\delta^4(P_B - P_K - P_X)|A(B \to KX)|^2 = f_K^2 \sum_X \int d^4y\ e^{-iy\cdot(P_B - P_K - P_X)}\\
&\times[|\alpha|^2 P_K^\mu P_K^\nu < B|\bar{b}\gamma_\nu(1-\gamma_5)q'|X >< X|\bar{q}'\gamma_\mu(1-\gamma_5)b|B >\\
&+|\beta|^2 < B|\bar{b}(1+\gamma_5)q'|X >< X|\bar{q}'(1-\gamma_5)b|B >]\\
&= f_K^2 \int d^4y\ e^{iy\cdot P_K}(|\alpha|^2 P_K^\mu P_K^\nu < B|[j_\nu^\dagger(0), j_\mu(y)]|B > +|\beta|^2 < B|[J^\dagger(0), J(y)]|B >),
\end{aligned} \tag{14}$$

where $j_\mu = \bar{q}'\gamma_\mu(1-\gamma_5)b$ and $J = \bar{q}'(1-\gamma_5)b$.

Computing the current commutators one obtains

$$\begin{aligned}
&\sum_X (2\pi)^4\delta^4(P_B - P_K - P_X)|A(B \to KX)|^2\\
&= -2f_K^2\left\{|\alpha|^2 P_K^\mu P_K^\nu(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta}) + |\beta|^2 g_{\alpha\beta}\right\}\\
&\times \int d^4y\ e^{iy\cdot P_K}[\partial^\alpha \Delta_{q'}(y)] < B|\bar{b}(0)\gamma^\beta(1-\gamma_5)U(0,y)b(y)|B > .
\end{aligned} \tag{15}$$

In the above we have assumed $m_{q'} = 0$ and used

$$\{q'(x), \bar{q}'(y)\} = i(\gamma\cdot\partial)i\Delta_{q'}(x-y)U(x,y), \tag{16}$$

with

$$U(x,y) = \mathcal{P}\exp[ig_s \int_y^x dz^\mu G_\mu(z)],$$
$$\Delta_{q'}(y) = -\frac{i}{(2\pi)^3} \int d^4k \; e^{-ik\cdot y} \epsilon(k^0)\delta(k^2), \tag{17}$$

where $U(x,y)$ is the Wilson link, G^μ is the background gluon field, and $\epsilon(x)$ satisfies $\epsilon(|x|) = 1$ and $\epsilon(-|x|) = -1$.

The matrix element $< B|\bar{b}(0)\gamma^\beta(1-\gamma_5)U(0,y)b(y)|B >$ which is equal to $< B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B >$ from parity consideration contains all information about initial bound state corrections. It is, however, difficult to completely evaluate it due to non-perturbative effects. In the following we attempt two calculations: one using light cone expansion, and the other using heavy quark effective theory.

A. Light Cone Expansion Estimates

In general one can decompose the matrix element, $< B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B >$, in the following form

$$< B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B > = 2[P_B^\beta F(y^2, y\cdot P_B) + y^\beta G(y^2, y\cdot P_B)], \tag{18}$$

where $F(y^2, y\cdot P_B)$ and $G(y^2, y\cdot P_B)$ are functions of the two independent Lorentz scalars, y^2 and $y\cdot P_B$.

Since we are interested in having large kaon energy $E_K > 2.1$ GeV and small invariant mass for the X, the dominant contribution to the y integration in Eq. (15) will be from the light cone region $y^2 \lesssim 1/E_K^2$, which suggestes that, as a good approximation, $< B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B > \approx 2P_B^\beta F(0, y\cdot P_B)$. This approximation is also supported by the fact that the function $\Delta_{q'}(y)$ has a singularity at $y^2 = 0$ while away from light cone it vanishes. Carrying out a Fourier transformation [8],

$$F(0, y\cdot P_B) = \int d\xi e^{-i\xi y\cdot P_B} f(\xi), \tag{19}$$

and inserting the above into Eq. (15), we arrive at

$$
\begin{aligned}
&\sum_X (2\pi)^4\delta^4(P_B - P_K - P_X)|A(B\to KX)|^2 \\
&= 8\pi f_K^2 (2|\alpha|^2 P_K^\alpha P_K\cdot P_B + |\beta|^2 P_B^\alpha) \\
&\times \int d\xi\ \delta[(\xi P_B - P_K)^2](\xi P_{B\alpha} - P_{K\alpha}) f(\xi).
\end{aligned} \tag{20}
$$

We finally obtain the decay distribution as a function of E_K

$$
\frac{d\Gamma(B\to KX)}{dE_K} = \frac{f_K^2}{2\pi m_B}\left(4|\alpha|^2 E_K^2 + |\beta|^2\right) E_K f(\frac{2E_K}{m_B}). \tag{21}
$$

Carrying out similar calculations, we obtain the differential decay rate for the $B\to K^*X$ decay

$$
\frac{d\Gamma(B\to K^*X)}{dE_{K^*}} = \frac{f_{K^*}^2}{2\pi m_B} 4|\alpha_*|^2 E_{K^*}^3 f(\frac{2E_{K^*}}{m_B}), \tag{22}
$$

where $\alpha_* = (G_F/\sqrt{2})\sum_{q=u,c} V_{qb}V_{qs}^*\tilde{A}^q$.

It is interesting to note that the same distribution function $f(\xi)$ appears in both $B\to KX$ and $B\to K^*X$ cases. It is also interesting to note that, in the approximation made in this section, the function $f(\xi)$ is the same as that in $B\to X\gamma$ [8] and semi-leptonic decays $B\to Xl\bar{\nu}$ [10]. These decays have been studied in details. Experiments in the future will measure the differential distributions for these decays and, therefore, provide detailed information about $f(\xi)$. We can use this information in the calculation to reduce error. One may also turn the argument around to use the decay modes discussed here to provide constraints on the form of the distribution function $f(\xi)$. Before the detailed experimental information becomes available, we have to make some theoretical modeling for our numerical analysis which will be discussed later.

B. Heavy Quark Effective Theory Estimates

We note that the simple expressions for the decay distributions in Eqs. (21) and (22) hold to leading order in light cone expansion. When higher order contributions are included the expressions will not be so simple. To have some idea about the sensitivity of the results

to other corrections, in the following we also estimate the corrections to the free b quark decay rates using heavy quark effective theory.

If the b quark is heavy, the decay products all have large energy and to a good approximation can be treated as free quarks. In that case, $U(0,y) \approx 1$ because the background gluon field can be approximated to vanish, we then have

$$< B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B > \approx < B|\bar{b}(0)\gamma^\beta b(y)|B > . \tag{23}$$

If the b quark is infinitively heavy, the above matrix element is simply given by $2P_B^\beta e^{-im_b v\cdot y}$, where v is the four velocity of the B meson satisfying $v^2 = 1$. Since the b quark has finite mass, there will be corrections. We now estimate the leading $1/m_b^2$ corrections following the procedure outlined in Ref. [9]. In the heavy quark effective theory, the $b(x)$ quark field can be expanded as

$$\begin{aligned}
b(x) &= e^{-im_b v\cdot x}\{1 + i\gamma\cdot D_T/(2m_b) + v\cdot D\gamma\cdot D_T/(4m_b^2) - (\gamma\cdot D_T)^2/(8m_b^2)\}h(x) + O(1/m_b^3)\\
&\quad + (\text{terms for anti-quark}),\\
D_T^\mu &= D^\mu - v^\mu v\cdot D,\\
D^\mu &= \partial^\mu - ig_s G^\mu(x).
\end{aligned} \tag{24}$$

Using the above expressions and keeping $1/m_b$ terms, we obtain [9]

$$\begin{aligned}
< B|\bar{b}(0)\gamma^\beta b(y)|B > &= 2m_B e^{-im_b v\cdot y}\{v^\beta - \frac{i}{6m_b}(2y^\beta + v\cdot y v^\beta)(\mu_\pi^2 - \mu_g^2)\\
&\quad - \frac{1}{8}(y^2 - (v\cdot y)^2)v^\beta\mu_\pi^2)\},
\end{aligned} \tag{25}$$

where

$$\begin{aligned}
\mu_g^2 &= \frac{1}{4m_B} < B|\bar{h}g_s G_{\mu\nu}\sigma^{\mu\nu}h|B >,\\
\mu_\pi^2 &= -\frac{1}{2m_B} < B|\bar{h}(iD_T)^2 h|B > .
\end{aligned} \tag{26}$$

We note that the expansion in Eq. (25) is different from the light cone expansion as can be seen from the above expression that some y^2 terms are kept. The expansion is truncated at

order $1/m_b$ in Eq. (25). The truncation of the $1/m_b$ expansion enforces the use of the quark level phase space, instead of the hadron level phase space.

Inserting the above expression into Eq. (13), we have

$$\Gamma(B \to KX) \approx \frac{f_K^2}{8\pi} m_b[|\alpha|^2 m_b^2(1+\frac{7}{6}\frac{\mu_g^2}{m_b^2}-\frac{53}{6}\frac{\mu_\pi^2}{m_b^2})+|\beta|^2(1-\frac{\mu_\pi^2}{2m_b^2}+\frac{\mu_g^2}{2m_b^2})]. \tag{27}$$

In the approximation made here, the distribution of E_K is a delta function with the peak at $E_K = m_b/2$.

Carrying out similar calculations, we obtain the decay rate for the $B \to K^* X$ decay,

$$\Gamma(B \to K^* X) \approx \frac{f_{K^*}^2}{8\pi} m_b |\alpha_*|^2 m_b^2 (1+\frac{7}{6}\frac{\mu_g^2}{m_b^2}-\frac{53}{6}\frac{\mu_\pi^2}{m_b^2}). \tag{28}$$

It is clear that in the limit of large m_b, that is $\mu_{\pi,g}^2/m_b^2 \to 0$, the result reduces to the free b quark decay $b \to K(K^*)q'$ result as expected.

The expressions for the decay rates, in the approximation we are working with, are simple, allowing easy analysis. In the case of the light cone expansion method, one needs to have detailed knowdege of distribution function $f(\xi)$ for numerical analysis. Although the detailed shape is not known, we do know some properties [8,10]. When integrating ξ from 0 to 1, $\int_0^1 d\xi f(\xi)$ must give 1 due to current conservation. If the decay can be considered to be a free b quark decay, then $U(0,y)=1$ because no background gluon field exists, and the b quark field is given by $b(y) = e^{-iy\cdot P_b} b(0)$, one obtains

$$f(\xi) = \delta(\xi - \frac{m_b}{m_B}). \tag{29}$$

We can also estimate the mean $<\xi> = \int_0^1 d\xi \xi f(\xi)$ and the variance $\sigma^2 = \int_0^1 d\xi \xi^2 f(\xi) - <\xi>^2$ using heavy quark effective theory. They are given by [10,8]

$$\begin{aligned} <\xi> &= \frac{m_b}{m_B}[1+\frac{5}{6m_b^2}(\mu_\pi^2-\mu_g^2)], \\ \sigma^2 &= \frac{\mu_\pi^2}{3m_B^2}. \end{aligned} \tag{30}$$

The small value for σ^2 implies that the distribution function is sharply peaked around m_b/m_B.

To go further we take the following parameterization for the distribution function [10]

$$f(\xi) = N\frac{\xi(1-\xi)^c}{[(\xi-a)^2+b^2]^d}, \tag{31}$$

where N is a normalization constant which guarantees $\int_0^1 d\xi f(\xi) = 1$. This function reduces to a δ-function with the peak at a as $b \to 0$. Comparing with Eq. (29), in this limit $a = m_b/m_B$. Once the parameters c and d are given, the parameters a and b can be fixed by comparing with $<\xi>$ and σ^2. Unfortunately we do not know the values for c and d at present. We will take c and d to be free parameters and vary them to see how the energy spectra of $K(K^*)$, branching ratios and CP asymmetries are changed.

IV. RESULTS AND DISCUSSIONS

We are now ready to present our numerical analysis. We will make theoretical predictions for the kaon energy spectra $d\Gamma/dE_{K^{(*)}}$, CP-averaged branching ratios and direct CP asymmetries defined as

$$\begin{aligned} Br_{ave}(B \to K(K^*)X) &= \frac{1}{2}[Br(B \to K(K^*)X) + Br(\bar{B} \to \bar{K}(\bar{K}^*)\bar{X})], \\ A_{CP}(B \to K(K^*)X) &= \frac{\Gamma(B \to K(K^*)X) - \Gamma(\bar{B} \to \bar{K}(\bar{K}^*)\bar{X})}{\Gamma(B \to K(K^*)X) + \Gamma(\bar{B} \to \bar{K}(\bar{K}^*)\bar{X})}. \end{aligned} \tag{32}$$

For the numerical analysis, we need to know the values for the parameters involved. Some of them are well determined. In our numerical calculations we will use the following values for the relevant parameters [11]: $m_b = 4.9$ GeV, $m_c = 1.5$ GeV, $m_s = 120$ MeV, $m_d = 4$ MeV, $m_u = 2$ MeV, $|V_{us}| = 0.2196$, $|V_{cb}| = 0.0402$, $|V_{ub}/V_{cb}| = 0.085$, $f_K = 160$ MeV, $f_{K^*} = 214$ MeV, $\alpha_s(M_Z) = 0.118$. We keep the CP violating phase γ to be a free parameter and vary it to see how the branching ratios and CP asymmetries depend on it.

The HQET parameter μ_g^2 can be extracted from the $B^* - B$ mass splitting: $\mu_g^2 = 3(m_{B^*}^2 - m_B^2)/4 \simeq 0.36$ GeV^2, while μ_π^2 is less determined. A calculation of QCD sum rules gives $\mu_\pi^2 = (0.5 \pm 0.2)$ GeV^2 [12], which is consistent with $\mu_\pi^2 = (0.45 \pm 0.12)$ GeV^2 from a recent lattice QCD calculation [13]. We will use $\mu_\pi^2 = 0.5$ GeV^2 for our numerical calculations.

In the case of light cone expansion, we also need to specify the distribution function $f(\xi)$. We will assume it to be the form given in Eq. (31). To have some idea how the kaon energy spectra, branching ratios, and CP asymmetries depend on the form of the distribution function, we consider two very different forms [14]: (i) preset $c = d = 1$, in that case $a = 0.9548$ and $b = 0.005444$ determined by the known mean value and variance of the distribution function; (ii) preset $c = d = 2$, in that case $a = 0.9864$ and $b = 0.02557$ determined by the same mean value and variance of the distribution function.

In Figs. 1-4, we show the kaon energy spectra in $B \to K(K^*)X$ decays computed in the light cone expansion approach, assuming $\gamma = 60°$. The solid and dashed curves correspond, respectively, to the parameter set (i) and (ii) for the distribution function. The kaon energy spectra are a discrete line at $E_{K^{(*)}} = m_b/2$ in free b quark decay approximation, which is not shown in the figures. We see that initial bound state effects stretch the spectra over the full kinematic range $0 \le E_{K^{(*)}} \le m_B/2$ and the kaon energy spectra depend strongly on the form of the distribution function. However, we note that all the spectra have more than 97% of events with $E_{K^{(*)}} > 2.1$ GeV. This implies that if the integrated branching ratios and CP asymmetries are measured with $E_{K^{(*)}} > 2.1$ GeV, the effects from the detailed shape of the distribution function are small.

We show the CP-averaged branching ratios, in Figs. 5-8, and the CP asymmetries, in Figs. 9-12, in $B \to K(K^*)X$ as a function of the CP violating phase γ. The solid curves are the results from the light cone expansion using the parameter set (i) for the distribution function, while the dashed curves are from the free b quark decay approximation. The initial bound state effects encoded in the distribution function almost cancel completely in the CP asymmetries in $B \to K^*X$, so that the solid and dashed curves coincide in Figs. 11 and 12. We find that the shifts in the branching ratios and CP asymmetries are negligible if the parameter set (ii) instead of (i) for the distribution function is used, indicating that both the branching ratios and the CP asymmetries are insensitive to the detailed shape of the distribution function.

One can clearly see from Figs. 5-8 that the differences between the solid and dashed

curves are small, about 2%. This implies that according to light cone expansion estimates, the initial bound state effects increase the CP-averaged branching ratios for $B \to K(K^*)X$ by about 2%, largely because the $B \to K(K^*)X$ phase space is used, which is larger than the $b \to K(K^*)q'$ phase space used in the free b-quark and heavy quark effective theory calculations. The branching ratios for $\bar{B}^0 \to K^-(K^{*-})X$ are sensitive to γ, varying from $0.53(0.25) \times 10^{-4}$ to $1.5(2.0) \times 10^{-4}$, whereas the branching ratios for $B^- \to \bar{K}^0(\bar{K}^{*0})X$ are not sensitive to γ, varying from $0.77(0.67) \times 10^{-4}$ to $0.84(0.74) \times 10^{-4}$. The above sensitivities to γ can be easily understood by noticing that the tree operators $O_{1,2}$ contribute to $\bar{B}^0 \to K^-(K^{*-})X$ decays but not to $B^- \to \bar{K}^0(\bar{K}^{*0})X$ decays when small annihilation contributions are neglected, resulting in strong dependence on $V_{ub}V_{us}^*$ for the former, but not for the latter.

For the same reasons, the CP asymmetries are expected to be much larger in $\bar{B}^0 \to K^-(K^{*-})X$ than in $B^- \to \bar{K}^0(\bar{K}^{*0})X$. The differences between the solid curves and dashed curves in Figs. 9 and 10 are very small, about 1%. This implies that according to light cone expansion estimates, the initial bound state effects increase the CP asymmetries in $B \to KX$ by about 1%. They do not affect the CP asymmetries in $B \to K^*X$. The CP asymmetries in $\bar{B}^0 \to K^-(K^{*-})X$ can be as large as 7%(14%), but very small ($< 1\%$) in $B^- \to \bar{K}^0(\bar{K}^{*0})X$, as expected.

The heavy quark effective theory estimates of the initial bound state effects are always to reduce the branching ratios at the level of 10% as can be seen from Eqs. (27) and (28) if $\mu_\pi^2 = \mu_g^2$ is used. In fact within the allowed range for μ_π^2 the initial bound state effects tend to reduce the branching ratios. The CP asymmetries are the same as those obtained by free b quark decay approximation.

The three estimates (free quark decay approximation, light cone expansion and heavy quark effective theory method) carried out here all give the same order of magnitudes for the branching ratios and CP asymmetries which are also the same order of magnitudes as those obtained in Ref. [2]. The initial bound state effects are at the order of 10% of the free b quark decay estimates. The differences between different methods may be viewed as

uncertainties in the estimates. The branching ratios are of order 10^{-4} and are within the reach of the B factories. The CP asymmetries in the neutral B modes $\bar{B}^0 \to K^-(K^{*-})X$ are large and can be measured at the B factories. When more data become available, one may obtain interesting information about hadronic effects and also information about the CP violating phase γ.

ACKNOWLEDGMENTS

This work was supported in part by NSC under grant number NSC 89-2112-M-002-058 and by the Australian Research Council.

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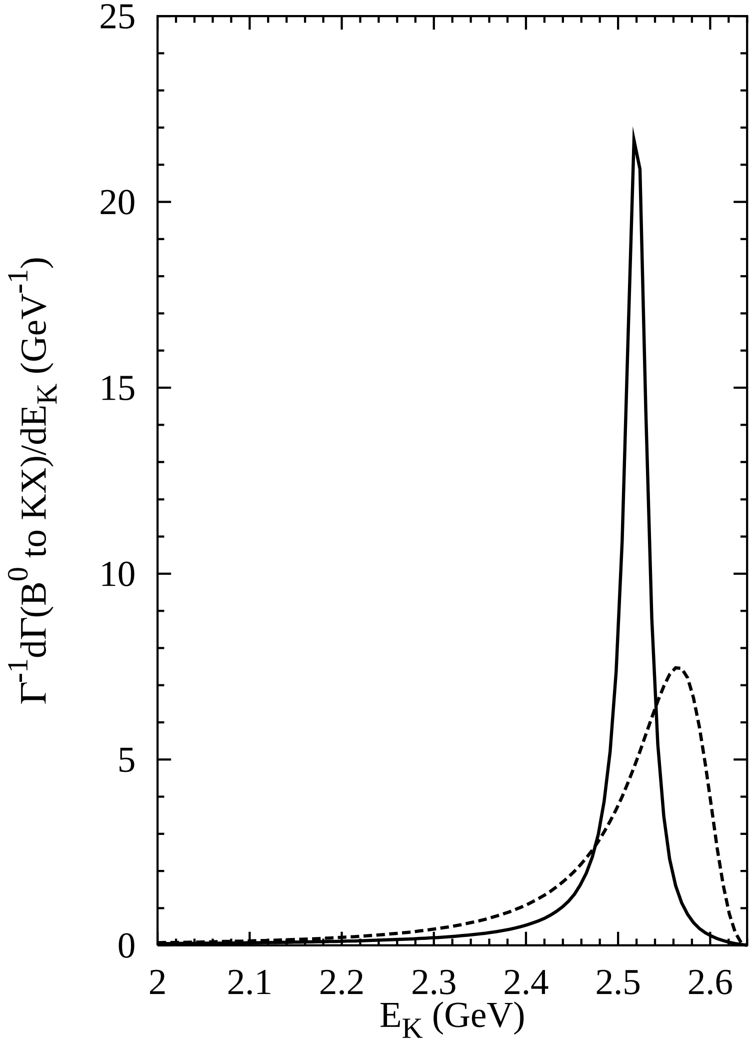

FIG. 1. Kaon energy spectrum in $\bar{B}^0 \to K^- X$. In Figs. 1-4, the solid curves are for (i) $c = d = 1$; the dashed curves are for (ii) $c = d = 2$.

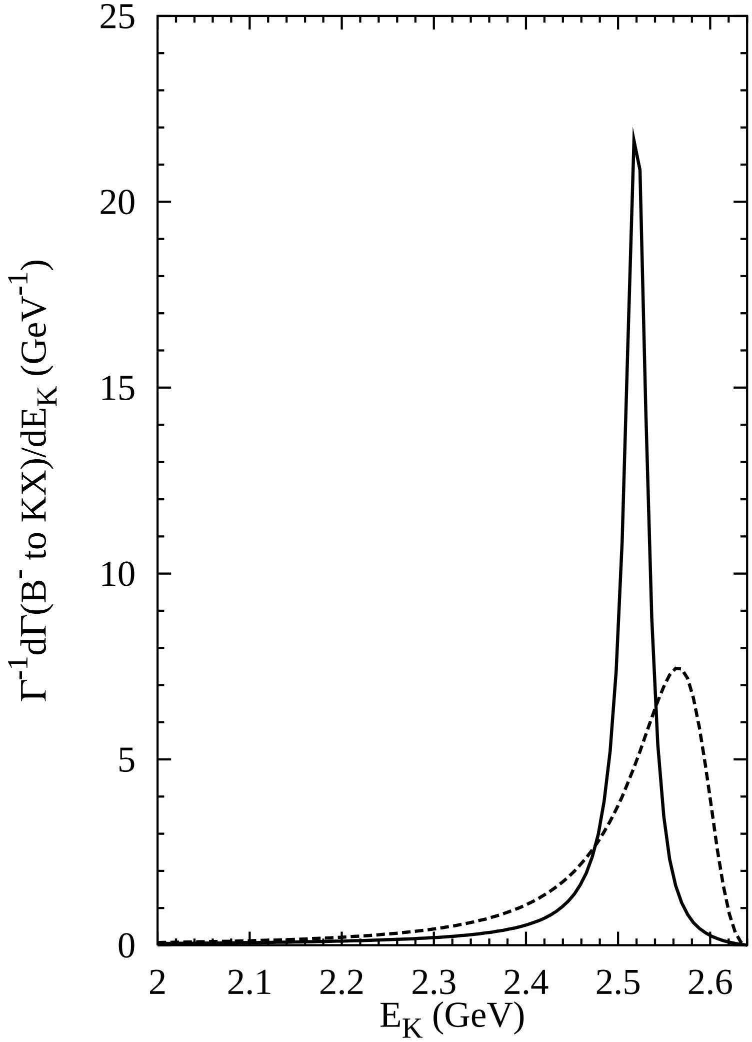

FIG. 2. Kaon energy spectrum in $B^- \to \bar{K}^0 X$.

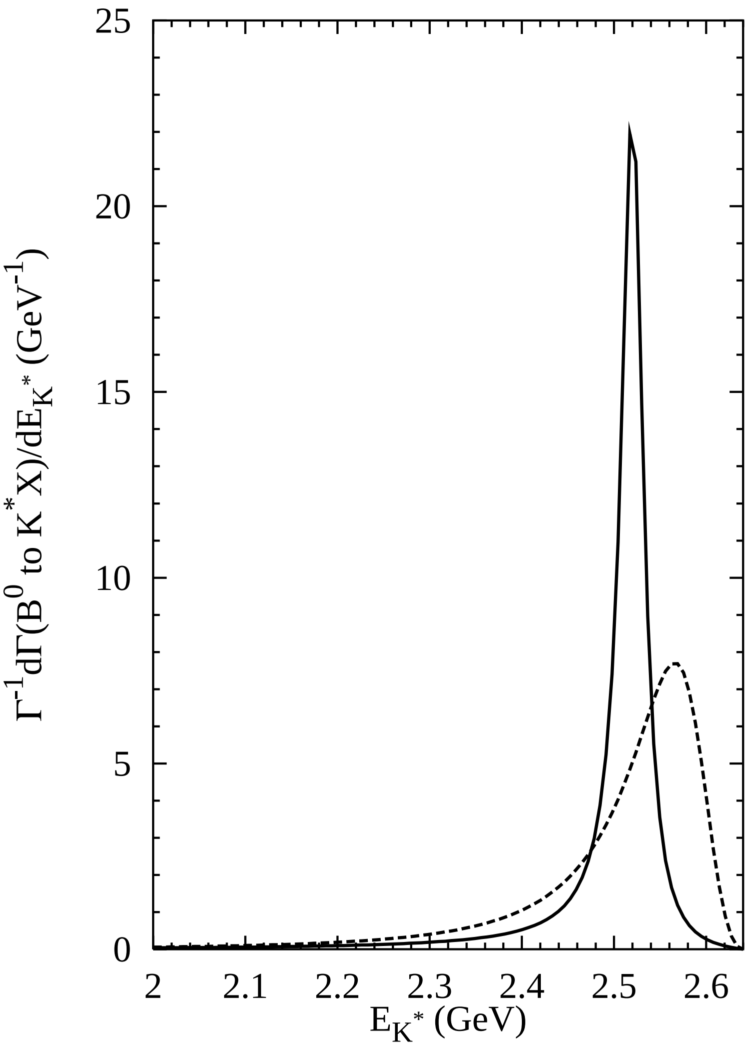

FIG. 3. Kaon energy spectrum in $\bar{B}^0 \to K^{*-}X$.

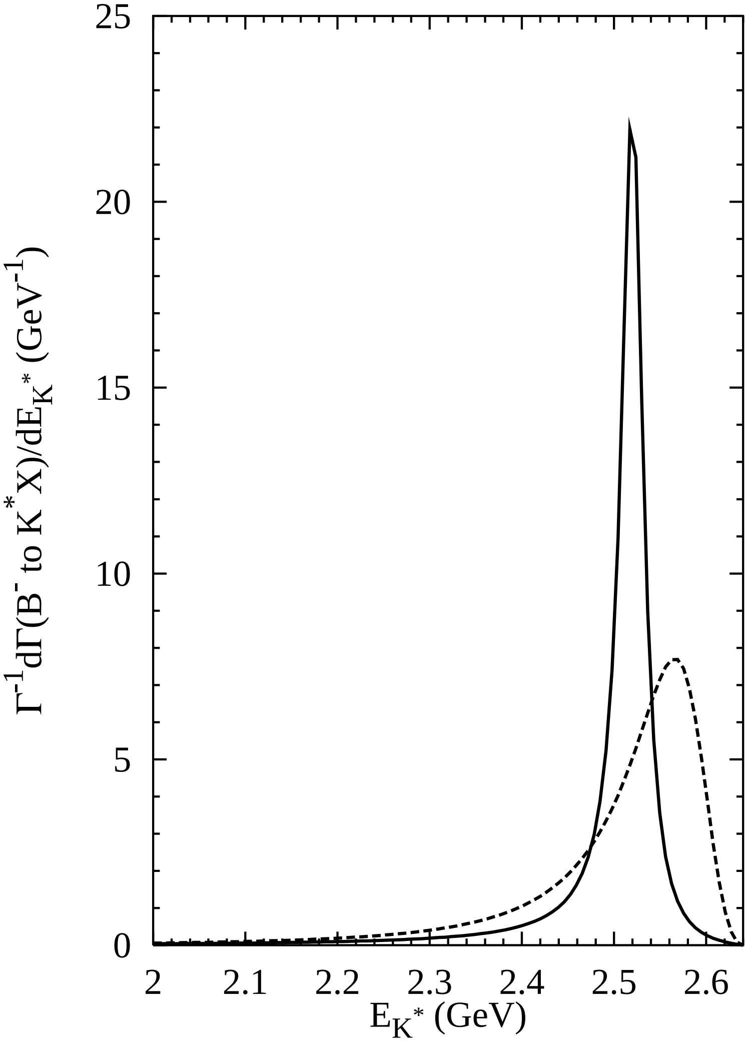

FIG. 4. Kaon energy spectrum in $B^- \to \bar{K}^{*0} X$.

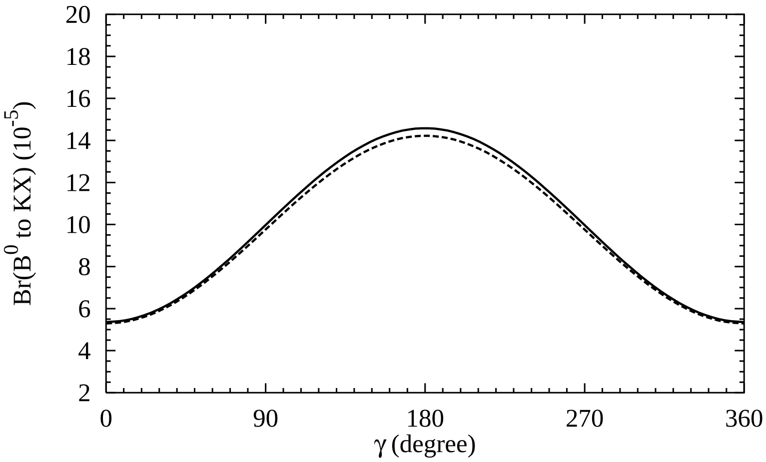

FIG. 5. CP-averaged branching ratio for $\bar{B}^0 \to K^- X$. In Figs. 5-10, the solid curves are for the ligh cone expansion with (i) $c = d = 1$; the dashed curves are for the free b quark decay approximation.

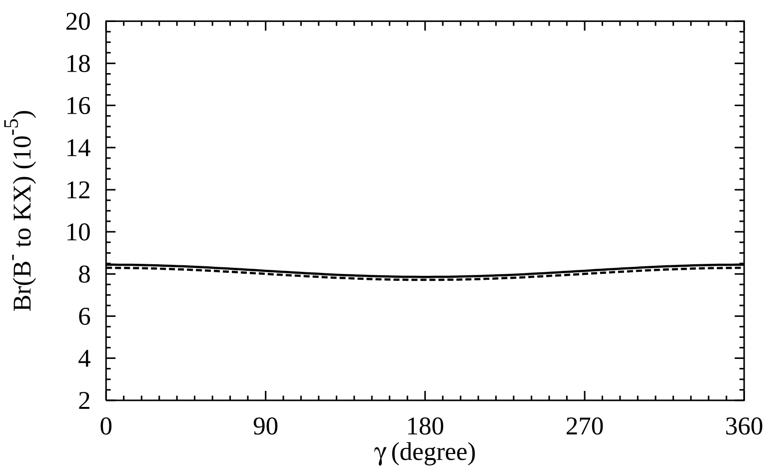

FIG. 6. CP-averaged branching ratio for $B^- \to \bar{K}^0 X$.

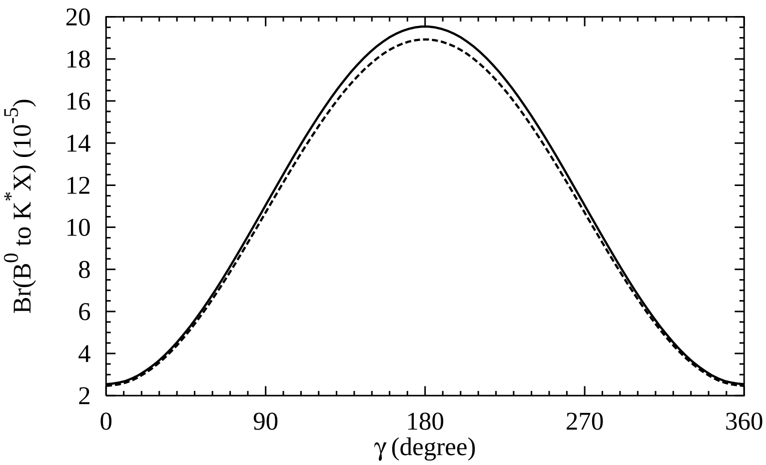

FIG. 7. CP-averaged branching ratio for $\bar{B}^0 \to K^{*-}X$.

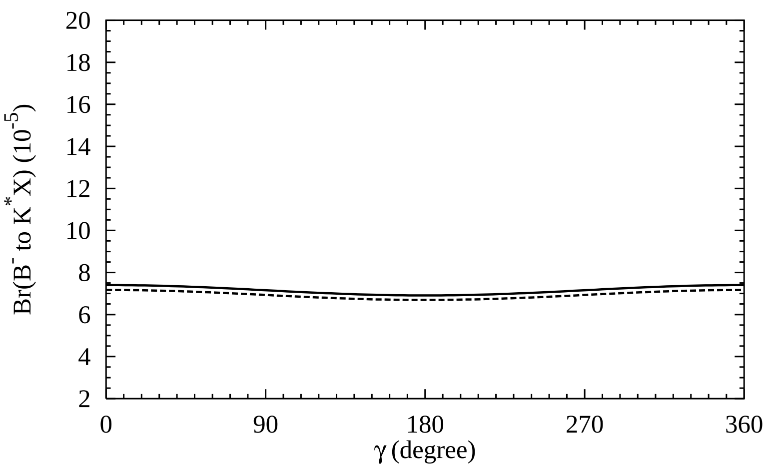

FIG. 8. CP-averaged branching ratio for $B^- \to \bar{K}^{*0}X$.

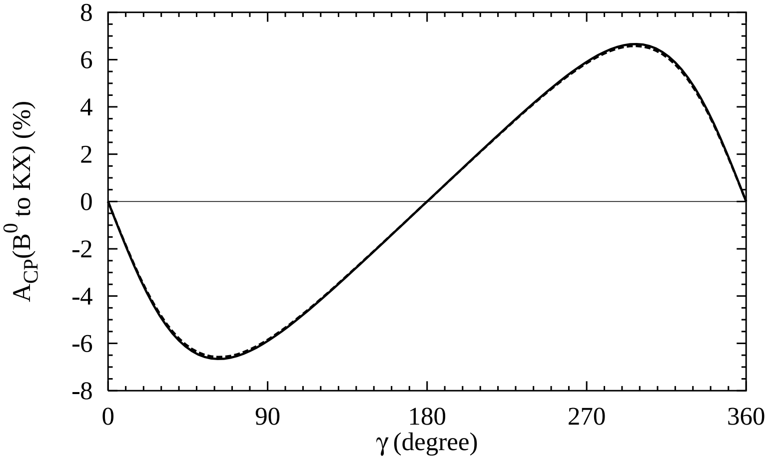

FIG. 9. CP asymmetry in $\bar{B}^0 \to K^- X$.

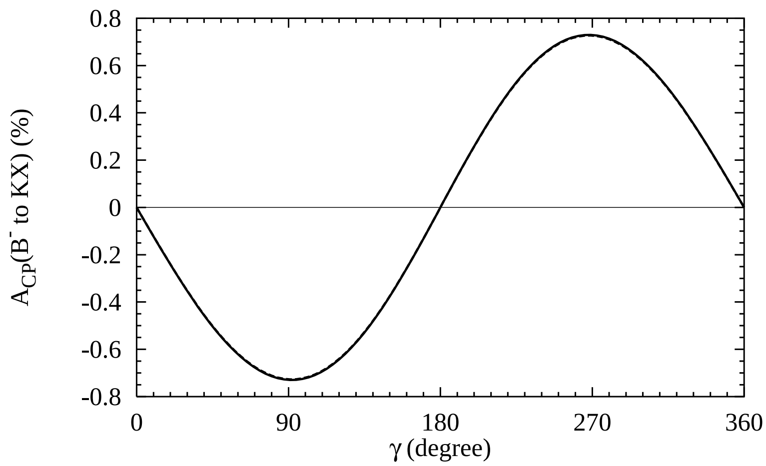

FIG. 10. CP asymmetry in $B^- \to \bar{K}^0 X$.

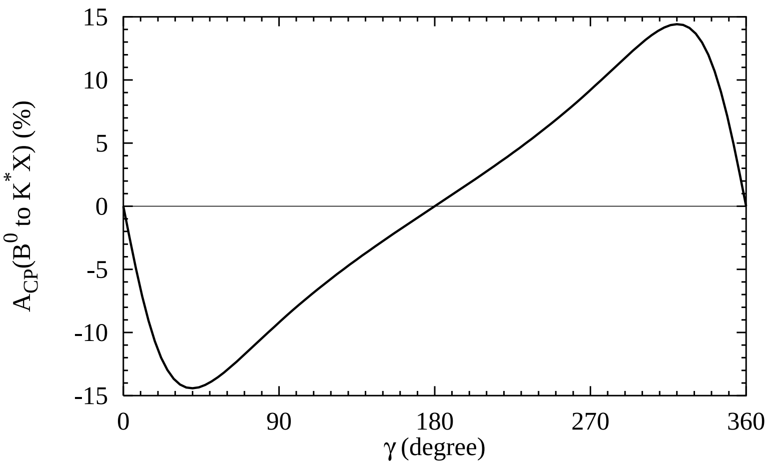

FIG. 11. CP asymmetry in $\bar{B}^0 \to K^{*-} X$.

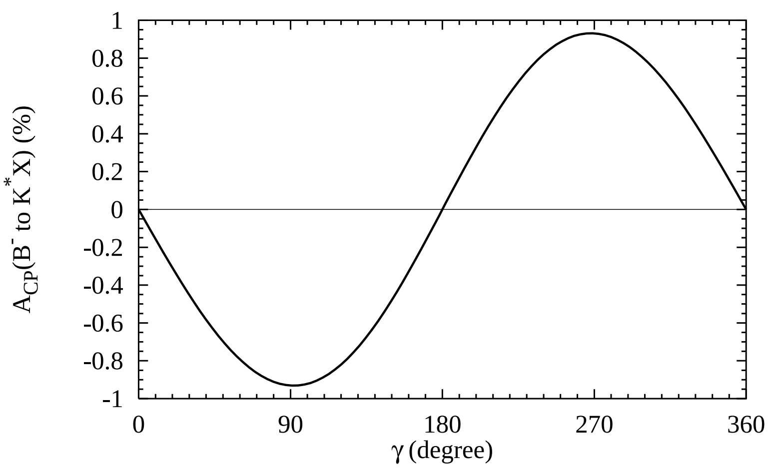

FIG. 12. CP asymmetry in $B^- \to \bar{K}^{*0} X$.